

# Evolutionary Analysis on Spatial Locality in the N-person Iterated Prisoner's Dilemma

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**Abstract.** The purpose of this paper is to consider the effects of the spatial locality on the evolution of cooperative behavior in the  $N$ -person iterated Prisoner's Dilemma ( $N$ -IPD) by focusing on two essentially distinct factors: *the scale of interaction* (which decides the neighboring members playing the  $N$ -person games) and *the scale of reproduction* (which decides the neighboring candidates for an offspring in each cell). We conducted the evolutionary experiments of strategies for one-dimensional  $N$ -IPD game with various settings of these two factors. Experimental results revealed that these two factors bring qualitatively different effects to the emergence of cooperative behavior. Furthermore, we investigated the dynamics of the evolution of spatial locality in  $N$ -IPD. When we introduced the evolution of the scale of interaction into our model, the dynamic evolution of the scale of interaction through generation facilitated the emergence of the global cooperation when the scale of reproduction was relatively small.

## 1 Introduction

Iterated Prisoner's Dilemma (IPD) has been widely studied for evolution of cooperation in various scientific fields. The evolutionary dynamics depends not only on the payoff matrix of IPD but also the expression or structure of interactions among agents as this game theoretic situation is based on the interactions between individuals. Therefore, many studies extended the structure of interactions so as to extend our knowledge about the evolution of cooperation.

In particular, spatial locality has been often introduced into evolutionary models as a more realistic and biological generalization of interactions. It is commonly said that spatial models bring about the higher degree of diversity in comparison with non-spatial models. Nowak and May adopted a two-dimensional deterministic IPD model which consists of the following two extreme strategies: those who always cooperate or those who always defect [1]. They observed chaotically varying spatial arrays, in which both strategies persist in shifting patterns when the payoff value for temptation to defect is modest. Lindgren and Nordahl explored the rule spaces of spatial games using more general strategies of various memory length which includes an open-ended evolution of the memory length

[2]. They also found that the spatial dynamics gives rise to a large variety of behavior even inside the Prisoner’s Dilemma region.

Besides, it is also said that cooperative strategies can easily occupy the population in the spatial IPD models compared to the non-spatial models, because cooperative strategies can make a cluster of own strategies while defect-oriented strategies can not. Grim reported that a strategy, which is twice as generous as the “GTFT” (cooperates on the first round, cooperates with probability 1/3 when the opponent defected on previous round, otherwise does whatever its opponent did) turned out to be optimal with a two-dimensional spatialization of a stochastic IPD model [3]. Suzuki and Arita investigated how local interactions among agents affected the benefit and the cost of learning through the evolutionary experiment of two-dimensional IPD game strategies of which the phenotypic plasticity were able to evolve [4], which was based on their previous study for interactions between evolution and learning [5]. They reported that the cooperative population was emerged through the Baldwin effect and how the learning behavior facilitates the evolution of cooperation is sensitive to the spatial locality.

Although many studies have been made on the spatial locality as mentioned above, most of them haven’t distinguished the essentially distinct two factors that concern the spatial locality. One is *the scale of interaction* which decides the neighboring members and group sizes playing the  $N$ -person games, and another is *the scale of reproduction* which decides the neighboring candidates for an offspring in each cell. One purpose of this paper is to consider the dynamics of the evolution of cooperative behavior in the spatial  $N$ -person iterated Prisoner’s Dilemma ( $N$ -IPD) by focusing on these two factors. There are few studies that explicitly separated these factors in their models. By altering the environmental parameters concerning these two factors in evolutionary experiments, we clarify that they have essentially different effects on the emergence and diversity of cooperative behavior in detail. Our second purpose is to discuss the evolution of the spatial locality in the context of  $N$ -IPD. In our real world, individuals usually select neighboring persons with whom they interact according to their own strategies, although many studies concerning the  $N$ -IPD fixed the neighboring members and group sizes as environmental factors. We conducted further evolutionary experiments on the condition that the scale of interaction can evolve as a part of strategy of each individual. There have been no other studies that various sizes of groups playing  $N$ -IPD are allowed to co-exist in the same generation.

## 2 The Model

### 2.1 $N$ -person Iterated Prisoner’s Dilemma

Iterated Prisoner’s Dilemma (IPD) is a two-person non-zero-sum game, which is widely adopted in various studies for the evolution of cooperative behaviors in social systems. In this paper, we consider the  $N$ -person generalizations of IPD because it can express a more realistic situation than two-person game [6].  $N$ -person game is carried out as following procedures:

1.  $N$  players independently choose actions from Cooperate (C) or Defect (D) without knowing the other players' choices.
2. Each player obtains the score according to the payoff functions  $C(c) = c/N$  for cooperation or  $D(c) = (c + k)/N$  for defection, where  $c$  is the number of cooperators and  $k$  is the total cost for cooperation ( $1 < k < N$ ). We term this procedure "round" (see Figure 1).
3. Players repeatedly play  $g$  rounds, then compete for higher average scores.

For example, this situation is concretely described as follows:

$N$  persons use the same public good and it costs  $k$  to maintain it in total per month regardless of  $N$ . They have accepted the unenforceable agreement that each individual contributes an equal share  $k/N$  every month. Furthermore, the benefits to each individual  $c/N$  are proportional to the number of the players who pay their share.

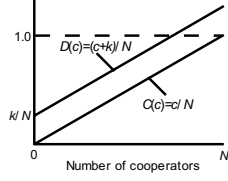
These payoff functions satisfy the following conditions which yields dilemma: 1) Defection is a dominant strategy because each player is better off by choosing defection than cooperation no matter how many of the other players choose cooperation (For  $1 \leq c \leq N$ ,  $D(c - 1) > C(c)$ ), 2) but if all cooperate they can get better scores than if all defect ( $C(N) > D(0)$ ). We adopt the normalized functions of which each value is divided by the number of players  $N$  so as to avoid the direct effect on possible score, which is caused by the difference of group size  $N$ .

Besides, these functions have a property that as the number of player  $N$  increases the cost for cooperation ( $k/N$ ) decreases. Bonacich classified various kinds of payoff functions for  $N$ -IPD into 5 types comparing their gains (the added benefit to each participant for universal cooperation over universal noncooperation) and temptations (the incentive for each individual not to cooperate) by focusing on their difference with various group sizes [7]. Our payoff functions described above belong to the type that gain increases and temptation decreases with group size. He pointed out that this was one of frequently encountered type of situation in which cooperative behavior can increase with group size, although there had been few studies on this type of functions.

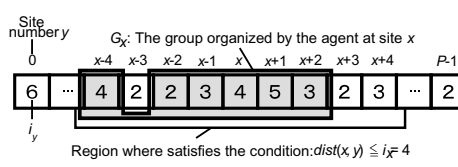
## 2.2 Strategies

In our model, each agent has  $N$ -person versions of "Tit-for-Tat" strategies (cooperates on the first round, whatever its opponent did on previous round) [8], which is described as two types of genetic parameter  $s$  and  $h$ .

The parameter  $s$  ( $0 \leq s < 1$ ) denotes the threshold which decides whether cooperate or defect according to the proportion of cooperators on previous round: If  $c/N \geq s$  then *cooperate* otherwise *defect*, where  $c$  is the number of cooperators on the previous round. For example, if 3 persons cooperated and 2 persons defected on the previous round in 5-person game, the player whose genetic parameter  $s$  equals to 0.2 cooperates on the next round.



**Fig. 1.** Payoff functions for N-IPD.



**Fig. 2.** The group organized by the agent at site  $x$ .

Though these strategies require the history of the previous actions to decide the next action, there is no history to decide actions on the first round. So, only on the first round, we assume that each player had chosen its own genetic parameter  $h$  (0: defection or 1: cooperation) as its action on the previous round. They decide first action referring to this “made-up” history.

### 2.3 Local Interaction

We assume an one-dimensional and troidal space which consists of  $P$  sites. Each site contains a single agent who plays  $N$ -IPD only with its neighbors. Here, we introduce the first factor of spatial locality: Each agent respectively organizes the group for  $N$ -IPD according to its genetic parameter  $i$  ( $2 \leq i \leq (P-1)/2$ ) which represent the scale of interaction of each agent. The group  $G_x$  organized by the agent at site  $x$  satisfies the following condition:

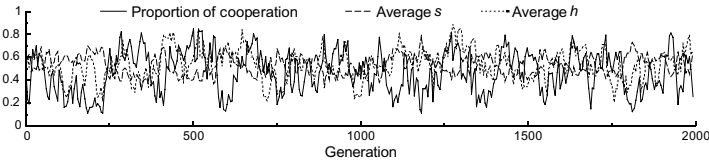
$$G_x = \{y | dist(x, y) \leq i_x \wedge dist(x, y) \leq i_y\}, \quad (1)$$

where  $i_x$  denotes the scale of interaction of the agent at site  $x$  ( $0 \leq x \leq P-1$ ) and  $dist(x, y)$  is the distance between site  $x$  and  $y$ . This equation expresses situation that the game must be conducted by mutual consent between the organizer and every other participant (Figure 2). Therefore, there can exist various group sizes for  $N$ -IPD in the same generation according to  $i$ s.

A different evolutionary approach concerning the spatial locality was conducted by Seo et al. [9]. They focused on the neighborhood size, which denotes the range of neighboring sites where  $N$  players are picked up randomly, and argued how it influences the emergence of cooperative coalition by using one-dimensional evolutionary model for  $N$ -IPD. They obtained interesting conclusion that the smaller the neighborhood size is, the more easily cooperation can emerge, even with a large group size. As their neighborhood size is distinct from our scale of interaction, we expect that the different type of correlation between the evolution of cooperation and the spatial locality will be observed.

### 2.4 Evolution

All genes ( $s$ ,  $h$  and  $i$ ) of agents are set randomly in the initial population. In each generation, we conduct iterated games in possible  $P$  groups ( $G_x, 0 \leq x \leq P-1$ )



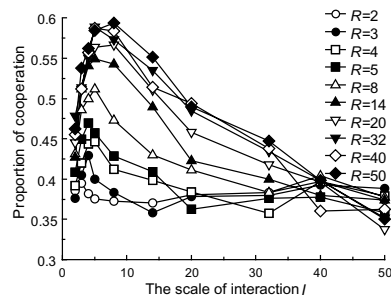
**Fig. 3.** Typical trial over 2000 generations ( $I = R = 8$  and  $k = 2.0$ ).

independently, under the condition in which the performed action can be changed by the noise with probability  $p_n$ . The average score of each agent over all rounds is regarded as a fitness value of each agent. The second factor concerning the spatial locality is adopted here: The agent that occupies each site in the next generation is selected from nearest  $2R+1$  agents by the “roulette wheel selection” according to the fitness, where the environmental parameter  $R$  corresponds to the scale of reproduction in our model. Mutation may replace each genetic parameter of offsprings with a randomly generated value according to the probability  $p_m$ .

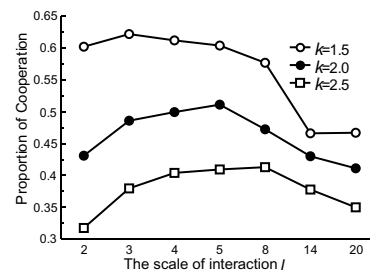
### 3 Experiments with the Fixed Spatial Locality

To analyze the basic dynamics of our system in detail, let us first consider the condition that all of genetic parameters  $is$  are always set to the same values  $I$  throughout generations. In other words, the environmental parameter  $I$  is given as the scale of interaction for all agents instead of  $is$  likewise the scale of reproduction  $R$ . Therefore all of groups for  $N$ -IPD organized by agents consist of nearest  $2I + 1$  neighbors. We conducted evolutionary experiments using the following values as the basic parameters:  $P=101$ ,  $p_m=0.05$ ,  $p_n=0.1$  and  $g = 100$ . All results are average over 5 trials every 5 generations.

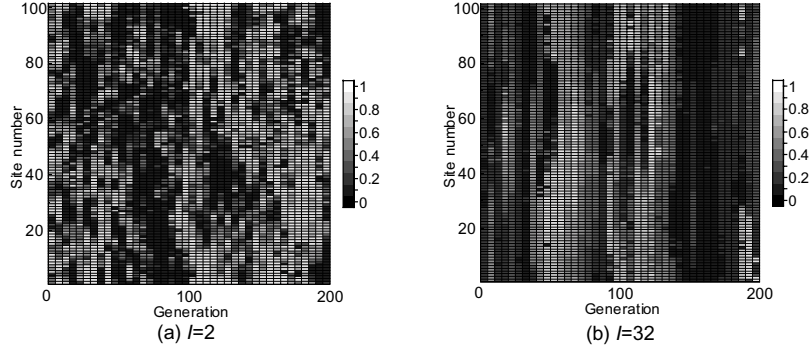
Figure 3 shows a typical trial over 2000 generations in case of  $I = R = 8$  and  $k = 2.0$ . The horizontal axis represents the generations, solid line shows the proportion of cooperation in all games, broken line represents the average



**Fig. 4.** The average proportion of cooperation with various  $I$  and  $R$  ( $k=2.0$ ).



**Fig. 5.** The average proportion of cooperation in case of  $k=1.5$ ,  $2.0$  and  $2.5$  ( $R=8$ ).

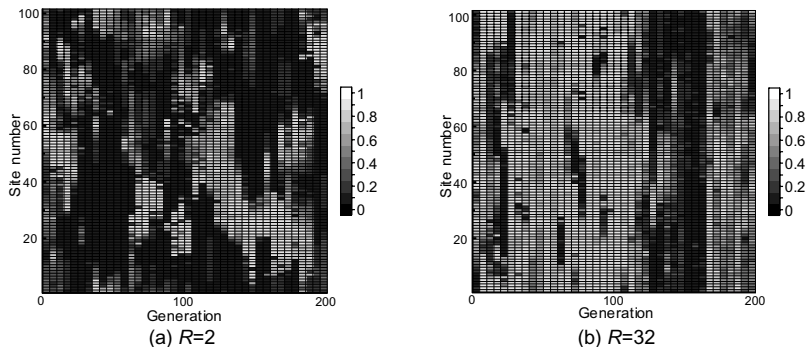


**Fig. 6.** The distribution of the proportion of cooperation ( $R=8$  and  $k=2.0$ ).

value of  $s$ , and dotted line shows the average value of  $h$  for each generation. Starting from randomly generated individuals, cooperative and defect-oriented strategies co-existed and partially occupied in turn. Population hadn't converged to a single strategy. The proportion of cooperators dynamically oscillated through the experiment. As a whole, population tends to be cooperative when the average  $s$  is small and the average  $h$  is large.

Figure 4 depicts the average proportion of cooperation over 2000 generations in various cases of  $I$  and  $R$ . The first thing we notice is that there are peaks of the proportion of cooperation. These peaks are due to the two positive and negative effects that facilitates the evolution of cooperation as  $I$  increases, which yield such a trade-offs. The positive effect is the property of the payoff functions as described before. That is, the more the number of players is in a group, the more easily the group establishes cooperation because the cost for cooperation gets lower. In fact, although the height of the peak decreases, it slightly shifts toward the larger condition of  $I$  as  $k$  increases as shown in Figure 5. It is because each group must compensate for larger cost for cooperation by extending group size.

The negative effect is derived from the diversity of proportion of cooperation between neighboring groups. The distributions of cooperative behavior in one-dimensional space over first 200 generations are shown in Figure 6 ((a) $I=2$ , (b) $I=32$ ,  $R=8$  and  $k=2.0$ ). The vertical axis represents the position on one-dimensional space where the organizer of each group was placed. The horizontal axis shows generations. The shading of each cell represents the proportion of cooperation in each organized group (black: 0.0, white: 1.0). When  $I$  is small, neighboring groups have different proportion of cooperation as shown in Figure 6 (a). This diversity of cooperators brings individuals diversity of fitness because the fitness obtained from each game is mainly depends on the proportion of cooperators. Oppositely, when the value of  $I$  is large, the proportion of cooperation tend to be close as shown in Figure 6 (b). It is because that neighboring groups have similar players as the number of players increases, then iterated games tend



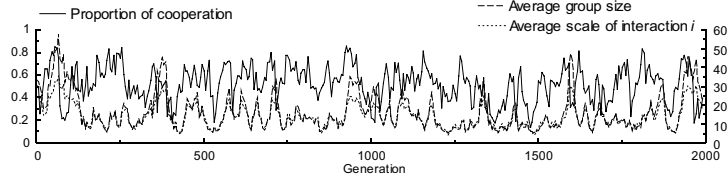
**Fig. 7.** The distribution of the proportion of cooperation ( $I=8$  and  $k=2.0$ ).

to end in similar result. (Extremely speaking, when  $I$  equals to the maximum value  $(P - 1)/2$ , all of  $P$  groups consist of the same  $P$  players.) Therefore if  $I$  is too large, multiple participation to neighboring games does not produce any selective force among neighboring individuals, which yields the negative effect for evolution of cooperation. These positive/negative effect bring the peak of the proportion of cooperation as  $I$  increase.

Also, we see from Figure 4 that the larger the scale of reproduction  $R$  is, the higher level cooperation is achieved. The impact of the scale of reproduction  $R$  on the distribution of the proportion of cooperation appears in Figure 7 ((a) $R=2$ , (b) $R=32$ ,  $I=8$  and  $k=2.0$ ). From these figures, when the value of  $R$  is relatively small, population is subdivided into small clusters of cooperative or defect-oriented groups. This is because that the local reproduction prevents any strategies from dominating the whole of population despite their characters. On the other hand, when  $R$  is relatively large, optimal strategies can rapidly and widely occupy the population. This global reproduction enables population to optimize the overall performance of population. Therefore the whole proportion of cooperation increases with  $R$ .

## 4 Evolution of the Spatial Locality

Next, we conducted the evolutionary experiment on the condition that the scale of interaction  $i$  is allowed to evolve, but all of the other settings and parameters are the same as those in the previous section. As members and size of each group are determined by the genetic parameter  $i$ , there can exist various group sizes in the same generation. Figure 8 shows a typical trial over 2000 generations in case of  $R = 8$ . The horizontal axis represents the generations. Solid line shows the proportion of cooperation in all games, broken line represents the average group size and dotted line also shows the average scale of interaction. From this figure, we realize that the evolutionary scenario did not converge to a discrete strategy, and the scale of interaction dynamically changed through the experiment.



**Fig. 8.** Typical trial over 2000 generations with evolution of the scale of interaction ( $R=8$  and  $k=2.0$ ).

Figure 9 also depicts the correlation between the proportion of cooperation and the average group size every 5 generations of another trial. This figure tells us that there is a soft correlation between these indices. Although the evolutionary scenario is quite difficult to predict as shown in Figure 8, it seems that this result is due to the fact that the large group size is advantageous for clusters of cooperative strategies if neighboring agents are also cooperative, because they can reduce the cost for cooperation as stated before. Thus, if the cooperative and locally interacting strategies appear in the population, the scale of interaction (or group size) occasionally tends to evolve to larger value, which leads the population to further evolution of cooperation. However, the cooperative population and larger group sizes also makes defect-oriented strategies easily invade because they can obtain large sucker's payoffs in such a population, then the evolutionary scenario puts back where it started.

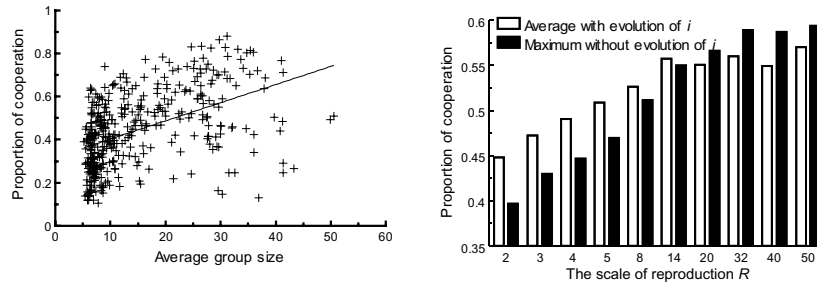
Table 1 shows the average  $i$  and group size throughout generation in comparison with the scale of interaction which maximize the proportion of cooperation without the evolution of  $I$ . We see from this table that dynamic evolution of  $i$  described above increased the average group sizes to nearly twice as large as the optimal group sizes without evolution of  $i$ , although there are a few exceptions.

Whether this dynamic evolution of the scale of interaction facilitates the emergence of cooperative behavior or not depends on the scale of reproduction. Figure 10 shows the proportion of cooperators through generation with various settings of the scale of reproduction  $R$  in comparison with the highest value of experiments without evolution of the scale of interaction as described in Figure

**Table 1.** The average group size with/without evolution of the scale of interaction

$R$	Average $i$	Average group size	$I$ (fixed group size)
2	16.9	19.6	40.0 (81.0)
5	15.9	18.7	4.0 (9.0)
8	15.6	18.5	5.0 (11.0)
14	16.9	20.8	5.0 (11.0)
32	18.4	23.7	5.0 (11.0)
50	18.6	24.3	8.0 (17.0)





**Fig. 9.** Correlation between the proportion of cooperation and group size ( $R = 8$ ,  $k=2.0$ ). **Fig. 10.** The proportion of cooperation with/without evolution of  $i$  ( $k=2.0$ ).

4. An important point to emphasize is that when the scale of reproduction  $R$  is less than around 14, the dynamic evolution of the scale of interaction facilitates higher level cooperation. Oppositely, the evolution of the scale of interaction restrains the evolution of cooperation when  $R$  is higher than around 14. Although this result is due to various factors, it is likely that the cooperative strategies which was emerged from the expansion of the scale of interaction as described above slightly increased the average proportion of cooperation in a locally reproducing population.

## 5 Conclusion

We have discussed how the spatial locality affects the emergence of cooperation in the evolution of  $N$ -person iterated Prisoner's Dilemma strategies by focusing on the essentially distinct two factors concerning the spatial locality: the scale of interaction and the scale of reproduction. Experiments with various environmental setups have revealed the following properties concerning the spatial locality: First, there exists a modest value of the scale of interaction (group size), with which the proportion of cooperation reaches its peak. Second, as the cost for cooperation increases, the overall proportion of cooperation decreases, but its peak slightly shifts toward the higher condition of the scale of interaction. Third, the larger the scale of reproduction is, the higher level cooperation is achieved. These results clearly show that each spatial locality has a quite different effect on the evolution of cooperation and both are important factors for the emergence of cooperative behavior.

Next, we conducted evolutionary experiments with evolution of the scale of interaction, where members and group size of each group for  $N$ -IPD were also decided by the interactions among agents. Evolutionary experiments have also brought about the following remarkable results: First, the average group size has a soft positive correlation with the proportion of cooperation. Second, the dynamic evolution of the scale of interaction causes larger average group size in comparison with the scale of interaction that maximize the proportion of cooperation without the evolution of the scale of interaction. However, it facilitates

the emergence of cooperative behavior only when the scale of reproduction is relatively small.

Locally reproducing population correspond to some of distributed evolutionary robotic systems such as embodied evolution[10], where agents evolve by locally exchanging genetic information. Our conclusion implies that introducing the mechanism which allows to select neighboring agents to interact with into genetic factor might increase the overall performance of the cooperative tasks.

Future works include introducing the evolution of the scale of reproduction into our models, adopting other types of  $N$ -IPD functions and conducting experiments with a deterministic evolutionary mechanism like a cellular automata.

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