Coevolution of game strategies, game structures and network structures

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Abstract. This paper aims at understanding the coevolutionary dynamics of game strategies, game structures and network structures of interactions. As a first approach, we constructed a coevolutionary model of game strategies and network modification strategies, in which individuals can evolve the game structure by developing new strategies that expand the existing payoff properties of Prisoner's Dilemma (PD) and Symmetric Coordination (SC). Evolutionary experiments showed that the dynamically evolving network brought about the emergence of an adaptive and mutually coordinating network from an isolated and defective population through a shift from a PD to a SC-type game structure, which bootstrapped the subsequent occurrence of adaptive coevolutionary cycles based mainly on a PD-type game structure.

Key words: Coevolution, evolutionary game, network structures, Prisoner's Dilemma, Symmetric Coordination, artificial life.

1 Introduction

Dynamics of mutual interactions between network topologies and states of the nodes are attracting much attention in various scientific and engineering fields [1, 2]. In evolutionary game theory, the spatial locality of interaction and reproduction has been regarded as a key factor for evolution of cooperation [4, 5, 6]. The evolution of strategies in evolving network structures is currently being investigated extensively [3]. This is because cooperative behaviors and network structures can coevolve by affecting their evolutionary dynamics mutually in real world situations – both physical and biological. This attention is also due to the recent interest in complex network structures in social relationships [8, 9].

Zimmerman and Eguiluz constructed a model in which the neighboring network structure of an individual can be changed according to the results of games with neighbors, in addition to the evolution of the Prisoner's Dilemma (PD) strategies [10]. They adopted a simple rule that the links between mutually defected individuals were rewired with other randomly selected individuals. They found that the emergence of the cooperative leader who had the largest payoff in the cluster of cooperative agents brought about the global cooperation. Pacheco *et al.* assumed a situation termed "active linking processes," in which there are different birth and death rates of links based on a combination of the strategies [11]. An important finding is that the effect of rapid evolution of the network structure could be interpreted as a transformation of the payoff matrix in an existing game. They also discussed it in the context of repeated games [12]. Tanimoto recently discussed a relationship between assortativity by degree of the evolved network and emerging cooperation in PD, showing that the weak (or strong) dilemma makes the assortativity of emerging networks positive (or negative) [13].

These studies were all based on the strong assumption that while every agent may have its own strategy for modifying its neighboring network, they all adopt the same fixed rule for rewiring. From this viewpoint, we constructed an evolutionary model in which each individual not only has a strategy for PD to play with its neighboring members on the network, but also has its own strategy for changing its neighboring structure of the network [14]. The behavior of this system was complex. We observed coevolutionary cycles of cooperating behaviors and the network structures, reflecting the dynamic aspect of the emergence and collapse of cooperative networks in a real world (see [14] for detailed analyses).

In this study, we focus on the evolution of game structures as another essential property of a real human society. Previous studies mainly discussed the evolution of strategies in the context of a unique 2×2 game such as PD or Snow Drift. However, in addition to choosing a strategy from the existing ones, it is also possible to evolve or expand the game structure by developing another new strategy of which the relationship with the existing ones reflects the existing game structure or constraint. For example, in a standard PD, agents may devise new strategies which bring about more beneficial cooperation, but they may also be exploited more heavily by existing strategies, due to some environmental constraints.

Our purpose is to clarify the coevolutionary dynamics of game structures and network structures of interactions. As a first approach, we expanded an evolutionary model of our previous work [14] to a version of a game, in which individuals can evolve the game structure by developing new strategies that expand the existing properties of Prisoner's Dilemma (PD) and Symmetric Coordination (SC). By starting experiments from the initial population of the minimal set of strategies, we discuss whether and how game strategies, game structures, and their network structures can coevolve by comparing the cases with and without evolution of network structures.

2 Expandable PD/SC game

We assumed a game, termed an Expandable PD/SC game, in which individuals can evolve the game structure by developing new strategies that expand the existing properties of Prisoner's Dilemma (PD) and Symmetric Coordination (SC).

In this model, we start the population with a minimal set of strategies. Let us call them strategy 1 and 2. There is a PD-type game structure between them. A new strategy can be developed from its adjacent strategies. For example, a new strategy, let us call it strategy 3, could be developed from the strategy 2 in the initial population. This expands the size of the payoff matrix, and another PD-type structure with expanded payoffs appears between the new strategy 3 and its adjacent strategy 2. At the same time, a SC-type structure appears between the new strategy 3 and its distant strategy 0. Similarly, a new PD structure appears between the strategy x and $x+1$ if a strategy $x+1$ is developed from existing strategies, and there appear SC structures between these new strategies and other distant strategies. Through this process, individuals can increase the size of the matrix indefinitely by developing an indefinite number of integer strategies. However, for simplicity, we limit its maximum value to M, which is large enough for discussion.

This situation could be interpreted as a kind of innovative evolution of decision making processes, technologies and so on, if we regard the difference in the strategy number also reflects the qualitative difference between such options or methods. Strategy $x+1$ can be interpreted as an improvement of the existing strategy x, and may have a conflict (PD) with it because they tend to share some social or physical resources. The succeeding strategy $x+2$ is a further improvement but more different from the strategy x, and tends to have less conflict (SC) with strategy x. However, it can be exploited by the existing option $x+1$.

Specifically, let us assume that the maximum strategy value in the current population is m. In this case, each individual can take an integer value $(1, 2, \cdots, m)$. If individuals A and B take strategies s_A and s_B respectively, the set of payoff values for these individuals p_A and p_B are determined by the following equation:

$$(p_A, p_B) = \begin{cases} (s_A, s_A) = (s_B, s_B) \text{ if } s_A = s_B, \\ (-S \times s_B, T \times s_A) \text{ if } s_A - s_B = 1, \\ (T \times s_A, -S \times s_B) \text{ if } s_B - s_A = 1, \\ \qquad (0, 0) \qquad\quad \textit{otherwise}. \end{cases} \tag{1}$$

Table 1 shows an example payoff matrix of Expandable PD/SC game when $T = 1.1$, $S = 0.3$ and $m = 5$. The properties of this matrix are summarized as follows:

- There is a structure of the PD game between two adjacent strategies. The strategy with a higher number can be interpreted as a cooperator, and the lower one can be interpreted as a defector in the context of a standard two-person prisoner's dilemma game. The parameter T determines the relative benefit of successful defection to that of mutual cooperation that is the same as the higher strategy value. The parameter S determines the relative cost of being exploited to the benefit of mutual defection that is the same as the lower strategy value.
- There is a structure of the SC game between two distant strategies. The individuals can obtain the payoff that is the same as their own strategy value only when they have the same strategy. Otherwise, their payoff becomes 0.

Table 1. An example payoff matrix of Expandable PD/SC game when m=5. Note that this payoff matrix is a snapshot during evolution of game structure when the maximum strategy value is 5. This matrix evolves its size gradually according to the invention of a new strategy.

player \ opponent	1	2	3	4	5
1	(1, 1)	(2.2, −0.3)	(0, 0)	(0, 0)	(0, 0)
2	(−0.3, 2.2)	(2, 2)	(3.3, −0.6)	(0, 0)	(0, 0)
3	(0, 0)	(−0.6, 3.3)	(3, 3)	(4.4, −0.9)	(0, 0)
4	(0, 0)	(0, 0)	(−0.9, 4.4)	(4, 4)	(5.5, −1.2)
5	(0, 0)	(0, 0)	(0, 0)	(−1.2, 5.5)	(5, 5)

(player's score, opponent's score)
$T > 1, S > 0$

– It is beneficial for the whole population to share the higher numbered strategy. At the same time, the degree of these payoff properties increases as the strategy values increase because of the increase in the absolute values of payoffs.

By using this expandable matrix, we can discuss how the evolution of game structure can affect the coevolution of game strategies and network structures.

3 Model

We constructed a coevolutionary model of strategies for the Expandable PD/SC game and network structures described above. This is basically based on our previous work on coevolution of strategies for Prisoner's Dilemma and network structures [14].

A population of N individuals are represented as nodes in the network and each (non directional) link between the two nodes represents that a mixed game will be conducted between the two individuals. Each individual has the information of four genes: g_{gs}, g_{na}, g_{ns} and g_{nd}. $g_{gs}(= 1, 2, \cdots M)$ directly encodes a strategy for the game. A set of g_{na}, g_{ns} and g_{nd} (0 or 1 respectively) represents the strategy for modifying its neighboring network structure as illustrated in Table 2.

Each step consists of the three phases defined as follows:

1. Each individual plays Expandable PD/SC games using g_{gs} against all neighboring (directly connected) individuals respectively, and obtains payoffs. At the same time, for each game, a fixed value σ is subtracted from the obtained payoff. σ is a constant which decides the relative difference in the payoff between the individual who played a game and the individual who did not play. The total payoff obtained in all participating games is taken as the fitness of each individual.

Table 2. A set of genetic information for network modification.

gene \ value	0	1
g_{na}	does nothing	creates a new link with a randomly selected individual
g_{ns}	does nothing	removes all links with the individuals with the same action
g_{nd}	does nothing	removes all links with the individuals with the different action

2. For each individual, if there are any neighboring individuals whose fitness is higher than that of the individual itself, the genetic information of the focal individual is replaced by that of the neighboring individual with the highest fitness. If there are more than one individual whose fitness tie as the highest among neighbors, an individual is selected randomly from them. Then, for each gene in all individuals, mutation occur with a small probability. E.g., for g_{gs}, a mutation occurs with probability p_{mg}. Such a mutation adds a randomly selected value from {-1, 1} to the current value of g_{gs}. If a generated value exceeds its domain, another mutation is operated on the original value again. As for g_{na}, g_{ns} and g_{nd}, a mutation may occur with probability p_{mn}, and inverts the corresponding genetic value. Note that the strategy of an isolated individual cannot be replaced by other strategies, but a mutation can occur. All updates of the genetic information occur at the same time.
3. Each individual modifies its neighboring network structure by using the results of games in phase 1 and its current network-modifying strategies. If $g_{ns} = 1$, the individual removes all links with the individuals whose action is the same as that of the individual itself. If $g_{nd} = 1$, the individual removes all links with the individuals whose action is different from that of the individual itself. In addition, if $g_{na} = 1$, the individual creates a new link with a randomly selected individual who was not connected with itself in phase 1.

This is repeated for G times.

4 Results

We conducted experiments using the following parameters: $N = 1000$, $M = 20$, $p_{mg} = p_{mn} = 0.02$ and G=5000. We adopted the condition for the payoff $T = 1.1$, $S = 0.3$ and $\sigma = 1.5$. The initial population was generated with initial values of g_{gs} that were randomly decided from 1 and 2; the genetic values of g_{na}, g_{ns} and g_{nd} were randomly assigned from $\{0, 1\}$. We adopted this initial condition to see whether and how dynamic evolutionary process can emerge from a simple and the least adaptive situation of Prisoner's Dilemma through the coevolution of game strategies, game structures and network structures.

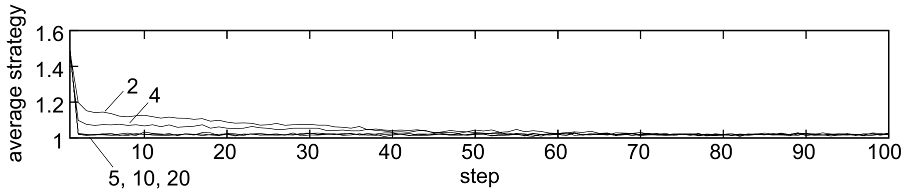

Fig. 1. The evolution of the average value of game strategies during initial 100 steps on regular networks with the degree D=2, 4, 10, 20 or 50.

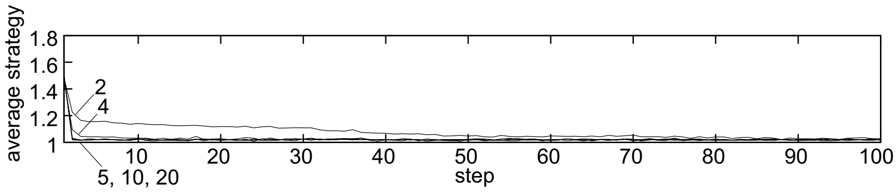

Fig. 2. The evolution of the average value of game strategies during initial 100 steps on random networks with the average degree D=2, 4, 10, 20 or 50.

4.1 Experiments with fixed network structures

First, we conducted experiments with fixed network structures. In these cases, we omitted the phase 2 in each step, so that the network structure was not modified by the individuals' network modification strategies through time steps. We adopted the following fixed structures: 1) one-dimensional regular networks in which each individual was connected with D (= 2, 4, 10, 20 or 50) neighboring individuals, 2) a random network with the average degree D (= 2, 4, 10, 20 or 50).

Fig. 1 shows typical examples of the evolution of the average value of game strategy through the initial 100 steps in cases of regular networks. The horizontal axis represents the step, and each line shows the average value of game strategy with the corresponding value of D. We see from this figure that the average strategy decreased very quickly from around 1.5, and converged to 1.0 in all cases. This means that adaptive populations did not emerge from the initial population of a PD-type game structure on regular networks. This seems to be due to the relatively high temptation to defect and small "sucker's payoff". We also see that the smaller D was, the slower the game strategy decreased. This is expected because that the higher spatial locality of small D values tended to retard the invasion by the defect-like strategy 1.

Random fixed network evolved similar to the cases of regular networks; the average strategy quickly decreased to 1.0 within 100 steps in the all cases of random networks, as shown in Fig 2.

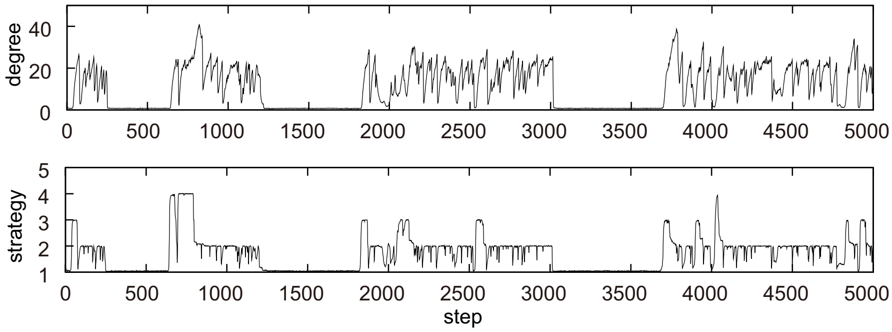

Fig. 3. An example evolutionary process of the average value of game strategies and the average degree over 5000 steps.

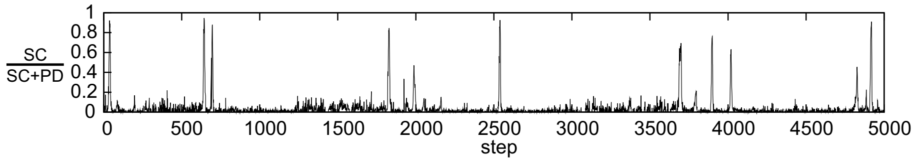

Fig. 4. An example evolutionary process of the proportion of the game structure.

As a whole, we can say that the population with fixed networks could not evolve adaptive relationship at all.

4.2 Experiments with coevolving game strategies and network structures

We conducted experiments with coevolving game strategies and network structures. We adopted the same parameters as the ones used in the previous section, and also adopted a random network with the average degree 2 as an initial network. Fig. 3 shows an example evolution of the average value of game strategies and the average degree over $G = 5000$ steps. Note that we observed the qualitatively similar phenomenon in every trial with this experimental condition. Contrary to the previous experiments, the average game strategy sometimes rapidly increased from around 1.0 and kept high value for several hundred steps, with a high average degree. Fig. 4 shows the evolution of the proportion of the game structures played between different game strategies in this experiment. The distribution of the game structures (PD and SC) between different strategies was calculated for each step. The value in Fig. 4 shows the proportion of games with SC structures among all games between different strategies. We see that it was basically small, which means that the games between different strategies were basically PD games. At the same time, we can also see that the proportion of SC games did increased rapidly for several times, together with the rapid increase in the average strategy shown in Fig. 3. Fig. 5 shows the evolution of the average

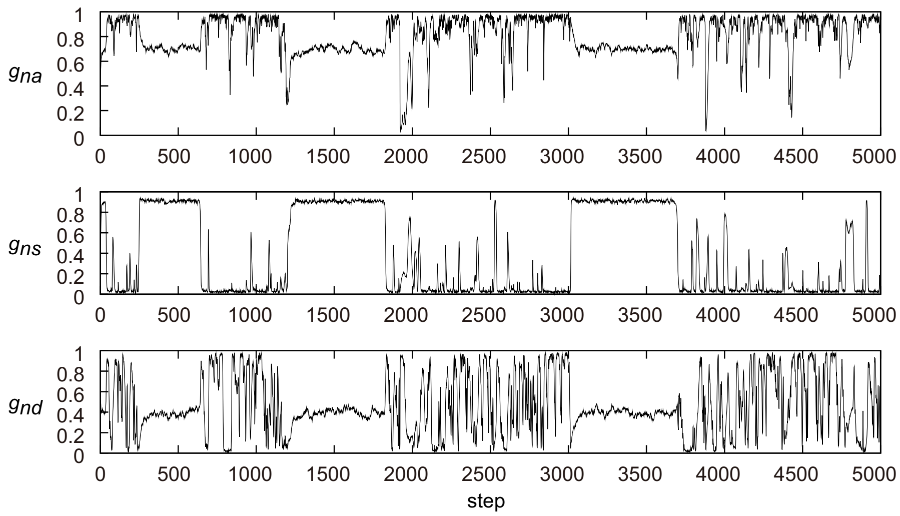

Fig. 5. An example evolutionary process of the average values of network modification strategies.

values of network modification strategies in the same experiment. Adaptive population was seen to emerge when both network modification strategies and game strategies could evolve. These results clearly show that the evolution of the game and network structures contributed to the emergence of adaptive relationships between individuals.

Detailed exploration of one example will help to understand the dynamics of evolution by focusing on the trajectory of the population on the two indices in Fig. 3, as is shown in Fig. 6. The cyclic evolutionary process of these indices is evident; each cycle was traversed basically in a counterclockwise manner. Fig. 7 illustrates a schematic image of evolutionary trajectory, although there were large fluctuations and exceptional moves in the trajectory. There were 6 evolutionary transitions in each cycle, in which can be summarized as follows:

(i) As observed around the 500th step in Fig. 3, the population sometimes stayed on the stable state in which both indices kept nearly the smallest. In this state, the population consisted of a large number of individuals with genes $(g_{gs}g_{na}g_{ns}g_{nd}) = (1110)$. They removed links with the same game strategy and connected to a randomly selected individual at each step. Because most of the individuals shared the same game strategy 1, they were repeating the rewiring process with randomly selected conspecifics every step. Thus, the network was sparsely and dynamically connected in this state. This state was stable in the sense that a mutant individual with the strategy 2 could not grow their clusters because of the high "sucker's payoff" and the high participation cost σ.

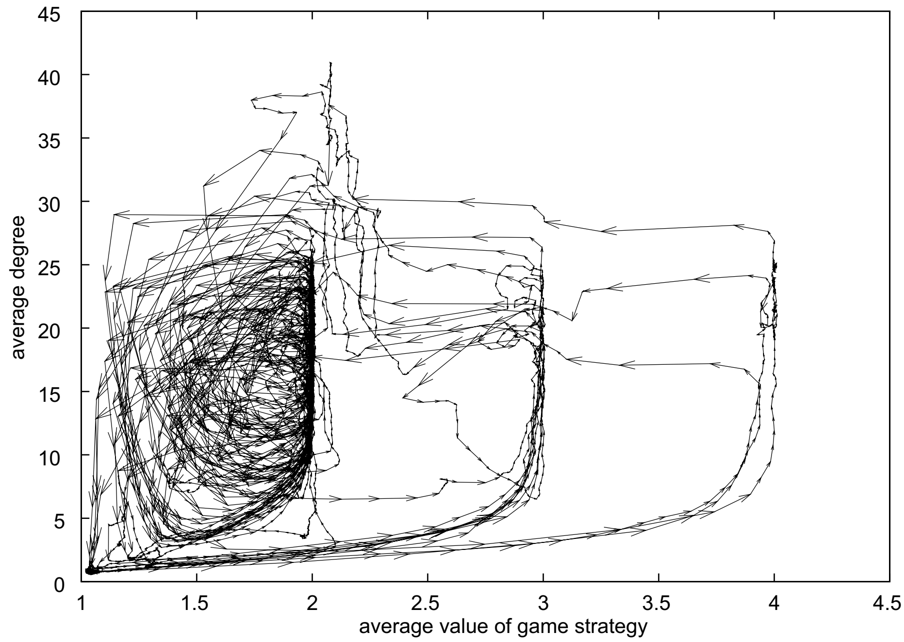

Fig. 6. An example evolutionary trajectory of the average value of game strategies and the average degree.

(ii) The dynamic property of the network in (i) sometimes created tiny clusters of individuals such as (3101) or (4101), and enabled them to invade into the population as observed at around the 650th step. Because their game strategy 3 or 4 played SC-type games with the game strategy 1, they could not be exploited by the dominant individuals. In addition, they maintained their links with the same strategies while removed the links with different strategies, and they also created a link with a random individual. This network modifying process enabled them to coordinate with each other and not to be exploited by the existing strategies. As a result, they could occupy the population quickly due to the mutual benefit of the change in the game structure and their adaptive network modification.

(iii) After such individuals occupied the population, they continued to increase their fitness by increasing their degree because of their high benefit of successful coordination. This gradually made the whole population highly connected. We also observed that g_{nd} gradually evolved from 1 to 0, which means that the individuals came to keep the links with different strategies. It reflects that keeping and increasing the degree was adaptive during this process. It is interesting that the population could keep both high average value of game strategies and high average degree, which is beneficial condition for the whole population but never observed in the cases without network evolution.

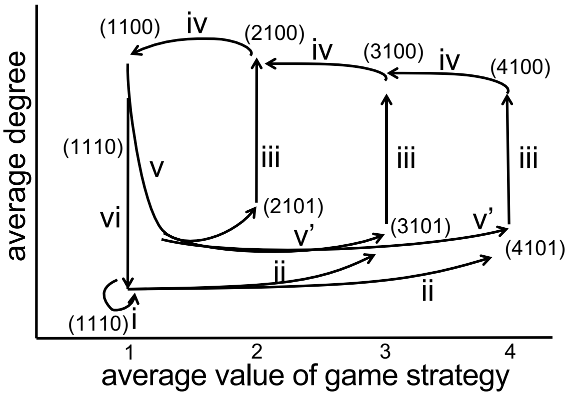

Fig. 7. A schematic image of evolutionary trajectory. Each arrow with the number corresponds to each evolutionary transition explained in the text. Each 4-length gene string ($g_{gs}g_{na}g_{ns}g_{nd}$) also represents the dominant genetic information at the corresponding state of the population on this graph.

(iv) However, the increase in the average degree changed the global property of population. When the average degree became sufficiently large, the smaller numbered game strategy than that of the dominant strategy by 1 quickly occupied the population, as observed at around the 800th step. They did not change the highly connected network structure, because they could successfully exploit the dominant strategy by using PD-type game structures in a relatively well-mixed population. This process occurred several times until the most population was occupied by the game strategy 1.

(v) When the most population was occupied by the game strategy 1, it became adaptive not to play a game. This is because that their net payoff from a game with each other was -0.5 $(= 1.0 - \sigma)$. It is smaller than the one when there was no game between individuals (0.0). Thus, the individuals (1110) that removed the links with the same dominant strategy but kept the ones with the different strategy began to occupy the population. This caused the rapid decrease in the average degree, which enabled the remaining small clusters of the individuals (2101) that killed the links with different strategies to grow again due to their cooperative benefit of a PD-type game structure. As a result, the population often repeated cycles composed of transitions (iii)-(iv)-(v), as observed at the 800-1200th step. This mechanism was basically similar to the ones observed in [14], but each cycle occurred in shorter time scale, which seems to be due to the relatively high mutation rates.
In addition, we sometimes observed the evolution of the higher numbered game strategy 3 or 4 through this process, as observed at around the 2100th or 2500th step (v' in Fig. 7). Although this seems to be due to the similar mechanism to the one in the transition (ii), it is interesting that the cyclic evolution facilitated the occurrence of these adaptive evolutionary transitions through the shift from a PD-type game structure to SC-types.

Clearly, these cyclic behaviors brought about more adaptive benefit for the whole population than the one in the least adaptive state observed in cases of fixed networks.

(vi) However, the whole population was sometimes completely occupied by the game strategy 1 during (iv) or (v), which made the population converge to (i).

As a whole, we can say that an emergence of a mutually coordinating network from an isolated and defective population through a shift from a PD to a SC-type game structure brought about the subsequent maintenance of adaptive coevolutionary cycles mainly based on a PD-type game structure.

5 Conclusion

We discussed whether and how game strategies and structures can coevolve with their network structures of interactions. As a first step, we conducted evolutionary experiments in which individuals can evolve the game structure by developing new strategies that expand the existing properties of Prisoner's Dilemma (PD) and Symmetric Coordination (SC), in addition to be able to modify their neighboring structure of network.

Evolutionary experiments based on an individual-based model in which the game strategy and the network modification strategy coevolve clearly showed that the dynamically evolving network brought about the emergence of an adaptive and mutually coordinating network from an isolated and defective population through a shift from a PD to a SC-type game structure, which bootstrapped the subsequent occurrence of adaptive coevolutionary cycles based mainly on a PD-type game structure. It should be emphasized that this process was not observed at all in experiments with several fixed networks, which implies that this kind of complex interactions among game strategies, game structures and network structures could be an important factor for maintenance of adaptive behaviors in a real human society.

Future work includes detailed analyses on the effects of parameters on the behaviors of the population, and experiments based on different kinds of game-theoretical situations.

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