

EVOLUTIONARY ANALYSIS ON SPATIAL LOCALITY IN N-PERSON ITERATED PRISONER'S DILEMMA

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The purpose of this paper is to consider the effects of spatial locality on the evolution of cooperative behavior in the N -person iterated Prisoner's Dilemma (N -IPD) by focusing on two essentially distinct factors: *the scale of interaction* (which decides the neighboring members playing the N -person games) and *the scale of reproduction* (which decides the neighboring candidates for an offspring in each cell). We conducted evolutionary experiments of strategies for one-dimensional N -IPD game with various settings of these two factors. Experimental results revealed that these two factors bring qualitatively different effects to the emergence of cooperative behavior. Furthermore, we investigated the dynamics of the evolution of spatial locality in N -IPD. When we introduced the evolution of the scale of interaction into our model, the dynamic evolution of the scale of interaction through generation facilitated the emergence of global cooperation when the scale of reproduction was relatively small. Experiments with the evolution of the scale of reproduction are also discussed.

Keywords: N -person IPD; spatial locality; evolution of cooperation.

1. Introduction

Iterated Prisoner's Dilemma (IPD) has been widely studied for evolution of cooperation in various scientific fields. The evolutionary dynamics depends not only on the payoff matrix of IPD, but also the expression or structure of interactions among agents, as this game theoretic situation is based on the interactions between individuals. Therefore, many studies extended the structure of interactions so as to extend our knowledge about the evolution of cooperation.

In particular, spatial locality has been often introduced into evolutionary models as a more realistic and biological generalization of interactions. It is commonly said that spatial models bring about the higher degree of diversity in comparison with non-spatial models. Nowak and May adopted a two-dimensional deterministic IPD model which consists of the following two extreme strategies: those who always cooperate or those who always defect.¹ They observed chaotically varying spatial arrays, in which both strategies persist in shifting patterns when the payoff value

for temptation to defect is modest. Lindgren and Nordahl explored the rule spaces of spatial games using more general strategies of various memory length which includes an open-ended evolution of the memory length.² They also found that the spatial dynamics gives rise to a large variety of behavior even inside the Prisoner's Dilemma region.

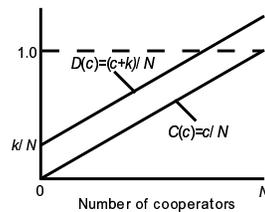
Besides, it is also said that cooperative strategies can easily occupy the population in the spatial IPD models compared to the non-spatial models, because cooperative strategies can make a cluster of own strategies while defect-oriented strategies cannot. Grim reported that a strategy, which is twice as generous as the "GTFT" (cooperates on the first round, cooperates with probability 1/3 when the opponent defected on previous round, otherwise does whatever its opponent did) turned out to be optimal with a two-dimensional spatialization of a stochastic IPD model.³ Suzuki and Arita investigated how local interactions among agents affected the benefit and the cost of learning through the evolutionary experiment of two-dimensional IPD game strategies of which the phenotypic plasticity were able to evolve,⁴ which was based on their previous study for interactions between evolution and learning.⁵ They reported that a cooperative population emerged through the Baldwin effect and how the learning behavior facilitating the evolution of cooperation is sensitive to spatial locality.

Although many studies have been made on the spatial locality as mentioned above, most of them have not distinguished the essentially distinct two factors that concern the spatial locality.⁶ One is *the scale of interaction* which decides the neighboring members and group sizes playing the N -person games, and the other is *the scale of reproduction* which decides the neighboring candidates for an offspring in each cell. One purpose of this study is to consider the effects of these two factors on the dynamics of the evolution of cooperative behavior in the spatial N -person iterated Prisoner's Dilemma (N -IPD). There are few studies that explicitly separate these factors in their models. By altering the environmental parameters concerning these two factors in evolutionary experiments, we clarify that they have essentially different effects on the emergence and diversity of cooperative behavior in detail. Our second purpose is to discuss the evolution of the spatial locality in the context of N -IPD. In our real world, individuals usually select neighboring persons with whom they interact according to their own strategies, although many studies concerning the N -IPD fixed the neighboring members and group sizes as environmental factors. We conducted further evolutionary experiments on the condition that the scale of interaction/reproduction can evolve as a part of the strategy of each individual.

2. The Model

2.1. N -person Iterated Prisoner's Dilemma

Iterated Prisoner's Dilemma (IPD) is a two-person non-zero sum game, which is widely adopted in various studies for the evolution of cooperative behaviors in social

Fig. 1. Payoff functions for N -IPD.

systems. In this paper, we consider the N -person generalizations of IPD because it can express a more realistic situation than two-person game.⁷ The N -person game is carried out as the following procedure:

- (1) N players independently choose actions from Cooperate (C) or Defect (D) without knowing the other players' choices.
- (2) Each player obtains the score according to the payoff functions $C(c) = c/N$ for cooperation or $D(c) = (c + k)/N$ for defection, where c is the number of cooperators and k is the total cost for cooperation ($1 < k < N$). We term this procedure "round" (see Fig. 1).
- (3) Players repeatedly play g rounds, then compete for higher average scores.

For example, this situation is concretely described as follows:

N persons use the same public good and it costs k to maintain it in total per month regardless of N . They have accepted the unenforceable agreement that each individual contributes an equal share k/N every month. Furthermore, the benefits to each individual c/N are proportional to the number of the players who pay their share.

These payoff functions satisfy the following conditions which yield the dilemma:

- (1) defection is a dominant strategy because each player is better off by choosing defection than cooperation no matter how many of the other players choose cooperation (for $1 \leq c \leq N$, $D(c - 1) > C(c)$),
- (2) but if all cooperate they can get better scores than if all defect ($C(N) > D(0)$).

We adopt the normalized functions of which each value is divided by the number of players N so as to avoid any direct effect on possible score caused by the difference of group size N .

Besides, these functions have a property that as the number of player N increases the cost for cooperation (k/N) decreases. Bonacich *et al.* classified various kinds of payoff functions for N -IPD into 5 types comparing their gains (the added benefit to each participant for universal cooperation over universal non-cooperation) and temptations (the incentive for each individual not to cooperate) by focusing on their difference with various group sizes.⁸ Our payoff functions described above

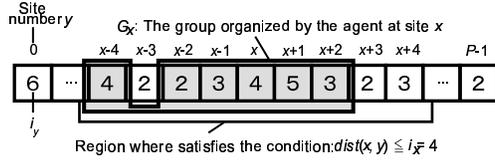


Fig. 2. The group organized by the agent at site x .

belong to the type that gain increases and temptation decreases with group size. He pointed out that this was one of the frequently encountered types of situation in which cooperative behavior can increase with group size, although there had been few studies on this type of functions.

2.2. Strategies

In our model, each agent has N -person versions of “Tit-for-Tat” strategies (cooperates on the first round, then does whatever its opponent did on previous round),⁹ which is described as two types of genetic parameter s and h .

The parameter $s \in [0, 1)$ denotes the threshold which decides whether the agent cooperates or defects according to the proportion of cooperators on previous round: if $c/N \geq s$, then *cooperate* otherwise *defect*, where c is the number of cooperators on the previous round. For example, if 3 players cooperated and 2 players defected on the previous round in 5-person game, a player whose genetic parameter s equals to 0.2 cooperates on the next round.

Though these strategies require the history of the previous actions to decide the next action, there is no history to decide actions on the first round. So, only on the first round, we assume that each player had chosen its own genetic parameter h (0: defection or 1: cooperation) as its action on the previous round. The players decide their first action referring to this “made-up” history.

2.3. Local interaction

We assume a one-dimensional and toroidal space which consists of P sites. Each site contains a single agent who plays N -IPD only with its neighbors. Each agent respectively organizes the group for N -IPD according to its genetic parameter $i \in [2, (P - 1)/2]$ which represent the scale of interaction of each agent. The group G_{i_x} organized by the agent at site x satisfies the following condition:

$$G_{i_x} = \{y | dist(x, y) \leq i_x \wedge dist(x, y) \leq i_y\} \tag{1}$$

where i_x denotes the scale of interaction of the agent at site x and $dist(x, y)$ is the distance between site x and y . This equation expresses the situation that the game must be conducted by mutual consent between the organizer and every other participant (Fig. 2). Therefore, there can exist various group sizes for N -IPD in the same generation according to i .

A different evolutionary approach concerning the spatial locality was conducted by Seo *et al.*¹⁰ They focused on the neighborhood size, which denotes the range of neighboring sites where N players are picked up randomly, and argued how it influences the emergence of cooperative coalition by using a one-dimensional evolutionary model for *N*-IPD. They obtained the interesting conclusion that the smaller the neighborhood size is, the more easily cooperation can emerge, even with a large group size. As their neighborhood size is distinct from our scale of interaction, we expect that a different type of correlation between the evolution of cooperation and spatial locality will be observed.

2.4. Local reproduction

All genes of agents are set randomly in the initial population. In each generation, we independently conduct iterated games in P possible groups (G_{ix} , $x \in [0, P-1]$), under the condition in which the performed action can be changed by the noise (mistake) with probability p_n . We define another genetic parameter $r \in [2, (P-1)/2]$ as the scale of reproduction in our model. The offspring that occupies the site x in the next generation is selected by the “roulette wheel selection” (in which the probability that an agent will be chosen as an offspring is proportional to its fitness) from neighboring group G_{rx} which satisfies the following condition:

$$G_{rx} = \{y | \text{dist}(x, y) \leq r_y\} \quad (2)$$

where r_y denotes the scale of reproduction of the agent at site y . Note that we did not adopt mutual consent among agents in the reproductive process because our model is based on unisexual reproduction. The fitness of the agent at site x is defined as $s_x \cdot (2r_x + 1)^{-\beta}$, where s_x is the average score of the agent at site x over all rounds and the parameter $\beta \in [0.0, 1.0]$ represents the cost for the participation in multiple reproductive processes. Mutation may replace each genetic parameter of offsprings with a randomly generated value according to the probability p_m .

3. Experiments with Fixed Spatial Locality

Firstly, we analyze the basic dynamics of our system on the condition that genetic parameters i and r are always set to the environmental parameters I and R throughout the generations. Therefore all of the groups for *N*-IPD organized by agents

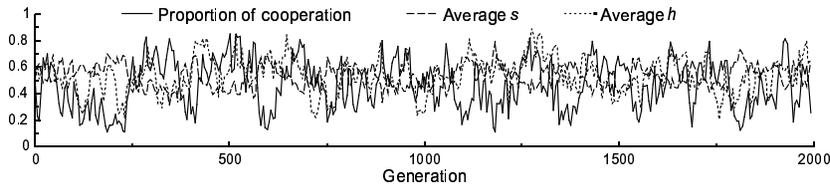


Fig. 3. Typical trial over 2000 generations ($I = R = 8$).

consist of the nearest $2I + 1$ neighbors and the agent that occupies each site in the next generation is selected from exactly $2R + 1$ neighboring sites. We conducted experiments using the following values as the basic parameters: $P=101$, $k = 2.0$, $p_m = 0.05$, $p_n = 0.1$, $g = 100$, $\beta = 0.0$. All results are averaged over 5 trials every 5 generations.

Figure 3 shows a typical trial over 2000 generations in case of $I = R = 8$. The horizontal axis represents the generations, solid line shows the proportion of cooperation in all games, broken line represents the average value of s , and dotted line shows the average value of h for each generation. Starting from randomly generated individuals, cooperative and defect-oriented strategies co-existed and the partially occupied in turn. The population had not converged to a single strategy. The proportion of cooperators dynamically oscillated through the experiment. As a whole, the population tends to be cooperative when the average s is small and the average h is large.

Figure 4 depicts the average proportion of cooperation over 2000 generations in various cases of I and R . The first thing we notice is that there are peaks of the proportion of cooperation. These peaks are due to the two positive and negative effects that facilitate the evolution of cooperation as I increases, which yield such a trade-off. The positive effect is the property of the payoff functions as described before. That is, the larger the number of players in a group, the more easily the group establishes cooperation because the cost of cooperation gets lower. In fact, although the height of the peak decreases, it slightly shifts toward the larger condition of I as k increases as shown in Fig. 5. It is because each group must compensate for the larger cost of cooperation by extending group size.

The negative effect is derived from the diversity of the proportion of cooperation between neighboring groups. The distributions of cooperative behavior in one-dimensional space over the first 200 generations are shown in [Fig. 6(a) $I = 2$, (b) $I = 32$ and $R = 8$]. The vertical axis represents the position on one-dimensional space where the organizer of each group was placed. The horizontal axis shows

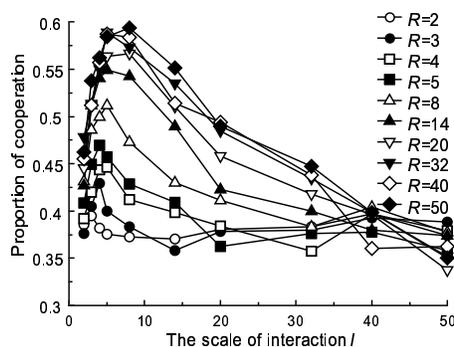


Fig. 4. The average proportion of cooperation with various I and R .

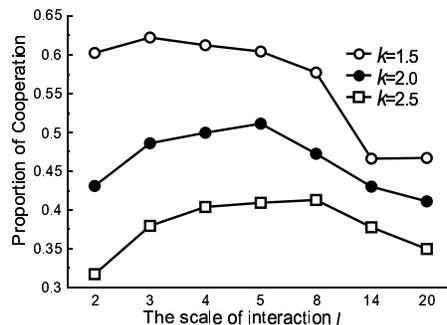


Fig. 5. The average proportion of cooperation in case of $k = 1.5, 2.0$ and 2.5 ($R = 8$).

generations. The shading of each cell represents the proportion of cooperation in each organized group (black: 0.0, white: 1.0). When I is small, neighboring groups have a different proportion of cooperation as shown in Fig. 6(a). This diversity of cooperators brings individuals diversity of fitness because the fitness obtained from each game mainly depends on the proportion of cooperators. By contrast, when I is large, the proportion of cooperation tend to be close as shown in Fig. 6(b). It is because that neighboring groups have similar players as the number of players increases, then iterated games tend to end in similar results. In the extreme, when I equals the maximum value $(P - 1)/2$, all of P groups consist of the same P players.) Therefore if I is too large, multiple participations in neighboring games does not produce any selective force among neighboring individuals, which yields the negative effect for evolution of cooperation. These positive/negative effects give rise to the peak of the proportion of cooperation as a function of I .

Also, we see from Fig. 4 that the larger R is, the greater the level of cooperation that is achieved. The impact of R on the distribution of the proportion

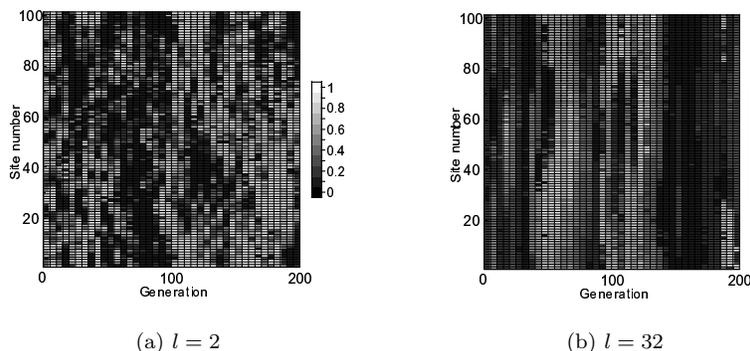


Fig. 6. The distribution of the proportion of cooperation ($R = 8$).

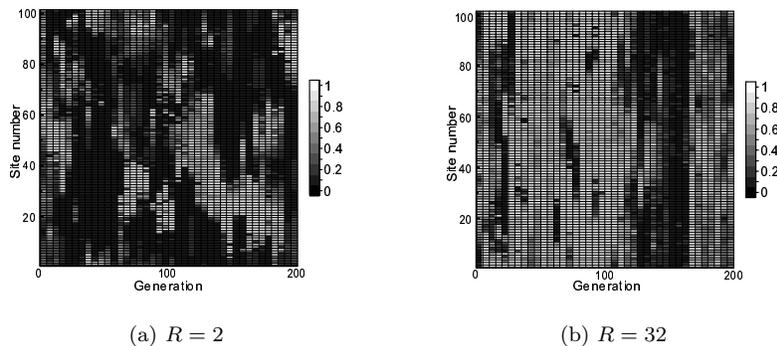


Fig. 7. The distribution of the proportion of cooperation ($I = 8$).

of cooperation appears in [Fig. 7(a) $R = 2$, (b) $R = 32$ and $I = 8$]. When R is relatively small, the population is subdivided into small clusters of cooperative or defect-oriented groups. This is because the local reproduction prevents any strategies from dominating the whole of the population despite their characters. By contrast, when R is relatively large, optimal strategies can rapidly and widely occupy the population. This global reproduction enables the population to optimize the overall performance of the population. Therefore the whole proportion of cooperation increases with R .

4. Evolution of the Spatial Locality

Next, we conducted an evolutionary experiment on the condition that the scale of interaction i is allowed to evolve, but all of the other settings and parameters are the same as those in the previous section. As members and size of each group are determined by i , there can exist various group sizes in the same generation. Figure 8 shows a typical trial over 2000 generations in case of $R = 8$ (fixed). The horizontal axis represents the generations. Solid line shows the proportion of cooperation in all games, broken line represents the average group size and dotted line also shows the average scale of interaction. We see that the evolutionary scenario did not converge to a discrete strategy and the scale of interaction dynamically changed through the

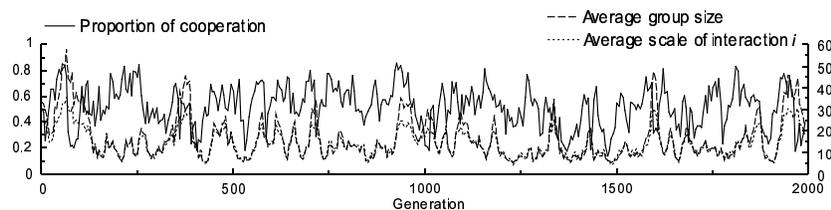


Fig. 8. Typical trial over 2000 generations with the evolution of the scale of interaction ($R = 8$).

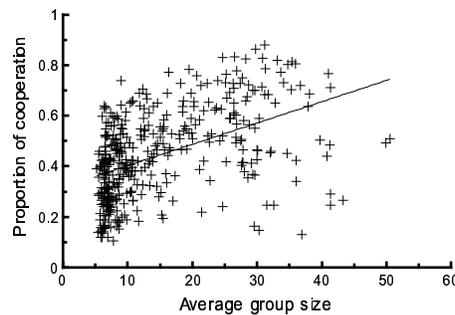


Fig. 9. Correlation between the proportion of cooperation and group size ($R = 8$).

experiment. In addition, the average group size evolved to nearly twice as large as the optimal group sizes without evolution of i (not shown).

Figure 9 also depicts the correlation between the proportion of cooperation and the average group size every 5 generations of another trial. This figure tells us that there is a soft correlation between these indices (in which the correlation coefficient is 0.48). Although the evolutionary scenario is quite difficult to predict as shown in Fig. 8, this seems to be due to the fact that larger group size is advantageous for clusters of cooperative strategies if neighboring agents are also cooperative, because they can reduce the cost for cooperation as stated before. Thus, if the cooperative and locally interacting strategies appear in the population, the scale of interaction (or group size) occasionally tends to evolve to larger value, which leads the population to further evolution of cooperation. However, the cooperative population and larger group sizes also make defection-oriented strategies easily invade because they can obtain many sucker's payoffs in such a population, then the evolutionary scenario returns where it started.

Whether this dynamic evolution of the scale of interaction facilitates the emergence of cooperative behavior or not depends on the scale of reproduction. Figure 10

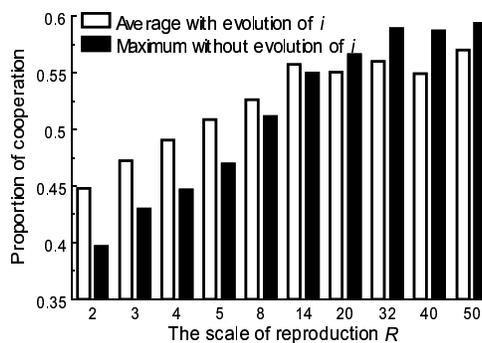


Fig. 10. The proportion of cooperation with/without evolution of i .

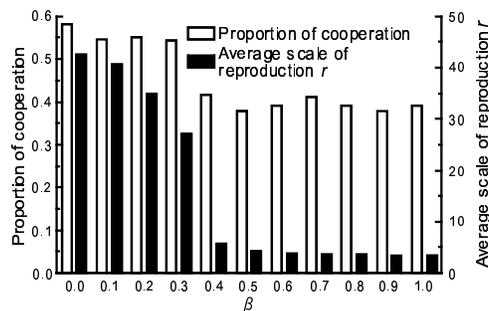


Fig. 11. Average proportion of cooperation and r in various cases of β ($I = 8$).

shows the average proportion of cooperation in various cases of R in comparison with the highest value of experiments without evolution of i as described in Fig. 4. An important point to emphasize is that when R is less than around 14, the dynamic evolution of i facilitates higher-level cooperation. Conversely, the evolution of i dampens the evolution of cooperation when R is higher than around 14. This seems to be due to the fact that cooperative strategies which emerged from expansions of i as described above slightly increased the average proportion of cooperation in a locally reproducing population.

Finally, we conducted experiments on the condition that the scale of reproduction r is allowed to evolve, but all of the other settings are the same as those in the previous section. Figure 11 shows the average proportion of cooperation and r over 2000 generations in case of $I = 8$ (fixed), $\beta = 0.0, 0.1, \dots, 1.0$. When $\beta = 0.0$, the average r evolves to the nearly maximum value. As β increases, the average proportion of cooperation gradually decreases but r drastically drops between $\beta = 0.3$ and 0.4 , then r converges to the minimum value when $\beta = 1.0$. This result implies that there exists a critical value between $\beta = 0.3$ and 0.4 which divides two opposite scenarios for evolution of r .

5. Conclusion

We have discussed how the spatial locality affects the emergence of cooperation in the evolution of N -person iterated Prisoner's Dilemma strategies by focusing on two essentially distinct factors concerning spatial locality: the scale of interaction and reproduction. Experiments have revealed the following properties: first, there exists a modest value of the scale of interaction, with which the proportion of cooperation reaches its peak. Second, as the cost for cooperation increases, the overall proportion of cooperation decreases, but its peak slightly shifts toward the higher condition of the scale of interaction. Third, the larger the scale of reproduction is, the higher-level cooperation is achieved. These results clearly show that each spatial locality

has a quite different effect on the evolution of cooperation and both are important factors for the emergence of cooperative behavior^a.

Next, we conducted experiments with the evolution of the scale of interaction/reproduction, where members and size of each group for *N*-IPD/reproduction were also decided by the interactions among agents. Experiments with the evolution of the scale of interaction have shown that the average group size has a soft positive correlation with the proportion of cooperation, and it facilitates the emergence of cooperative behavior when the scale of reproduction is relatively small. Experiments with the evolution of the scale of reproduction have also shown that the evolution of the scale of reproduction strongly depends on the cost for participation in multiple reproductive processes.

Locally reproducing populations correspond to the distributed evolutionary robotic systems such as embodied evolution,¹¹ where agents evolve by locally exchanging genetic information. Our conclusion implies that incorporating the properties which corresponds to the scale of interaction/reproduction into the evolutionary mechanism increases the overall performance of the cooperative tasks, and that the cost for reproduction is a crucial factor which decides the evolutionary scenario of systems.

Future works include further analysis on the evolution of the scale of reproduction, adopting other types of *N*-IPD functions and conducting experiments with a deterministic evolutionary mechanism like cellular automata.

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^aAll source codes will be available via <http://www2.create.human.nagoya-u.ac.jp/~reiji/n-ipd/>.

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